Quiz 9 (10pts)
Math 214 Section Q1 Winter 2010

Your name:_____ ID#:_____

Please, use the reverse side if needed.

1.(5 pts) Find the limit or show that it does not exist

$$\lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x+y}{x-y}.$$

Solution.

Let $y = kx$, then along this line the function equals

$$\frac{x+y}{x-y} = \frac{x+kx}{x-kx} = \frac{1+k}{1-k}.$$

Therefore, the limit is different along different directions (for example, if $k = 0$, it is 1, and if $k = 2$, it is -3). So, according to the two-paths test, the limit of the function does not exist.

2.(5 pts) Let $w = xy + x^2 - y^2$, $x = 2t + s^2$, $y = t^2 - s$. Use the Chain Rule to find $\dfrac{\partial w}{\partial s}$. Express the answer in terms of t and s.

Solution.

The Chain Rule gives $\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s}$

Now compute the partial derivatives in the above formula

$$\frac{\partial w}{\partial x} = y + 2x, \qquad \frac{\partial w}{\partial y} = x - 2y$$

$$\frac{\partial x}{\partial s} = 2s, \qquad \frac{\partial y}{\partial s} = -1$$

Then

$$\frac{\partial w}{\partial s} = (y+2x)2s+(x-2y)(-1) = (t^2-s+4t+2s^2)2s-(2t+s^2-2t^2+2s)$$

$$= 2st^2 - 3s^2 + 8ts + 4s^3 - 2t + 2t^2 - 2s.$$

It's OK if you don't simplify.